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## AN ANALYSIS OF SPATIAL CONFIGURATION AND ITS APPLICATION TO RESEARCH IN HIGHER EDUCATION

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## Abstract

An analysis of the spatial configuration of variables in a multivariate system is presented. The purpose of the analysis is to make clearer the relationships among the variables by locating them in a minimally-dimensioned space. Similarly, individuals are located in the smaller space and related to each other on the basis of the variables measured.

The analysis is then used to locate some colleges on a planar surface on the basis of variables given by Astin. In the configuration of colleges in the plane, a college is described in terms of its relative orientation to several educational aspects and the resulting single point location is suggested as a valuable alternative to profile analysis.

# AN ANALYSIS OF SPATIAL CONFIGURATION AND ITS APPLICATION TO RESEARCH IN HIGHER EDUCATION

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Measurement instruments in the social sciences are often multivariate. Rarely, however, are the variables independent. Typically more variables are used than the actual dimensionality of the data suggests. Such practice appears justified when the variables are measures of meaningful characteristics but the dimensions of a smaller dimensioned space are not similarly meaningful or conducive to direct measurement. Even when justified, the greater dimensionality increases the difficulty of clinical and research use of the data. Therefore, the specification of the variables within a minimally-dimensioned space would seem to provide a useful simplification.

The primary motivation for this analysis of spatial configuration is to provide a method for understanding relationships among the variables. The information necessary for such understanding is contained in the correlation matrix, but it is quite difficult to interpret the correlations in the matrix simultaneously. The analysis overcomes this difficulty by often achieving a visual representation of the variables which can be of considerable value in understanding the relationships among the variables.

While using dimension reduction techniques of factor analysis and multidimensional scaling, this analysis is not intended as a method for identifying a smaller number of variables in a system. Thus, the use of factor analysis to replace a large number of variables with a few factors has quite a different motivation from ours. This analysis reduces the dimensionality of the space in which the variables are imbedded but retains the variables. If there were too many variables in the system before the analysis, there will still be too many after the analysis. The purpose is rather to present the variables in a reduced space in which their relationships can be more easily conceived.

A secondary asset of the procedure is that it provides a representation of important aspects of some kinds of profile data with the result that these aspects are more easily and meaningfully evaluated than they are in profile form.

The first portion of the paper presents the mathematical formulation of the method. An illustration of its use and discussion of its advantages and disadvantages in application are presented in the second part of the paper. Readers interested in the application to data may prefer to start with the section beginning on page 5 and use the first section as a reference for the details of the procedure.

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The basic statistical technique used in this analysis of spatial configuration is the technique of principal components. First developed in general form by Hotelling (1933), principal components are linear combinations of variables with special variance properties. The first principal component is the normalized linear combination of the variables with maximum variance. The second is the normalized linear combination with maximum variation orthogonal to the first component. Stated generally, the i-th principal component is the normalized linear combination which maximizes the variance orthogonal to the first $i-1$ components (Anderson, 1958).

Another interpretation of principal components involves the definition of a subspace of an original space with certain optimal properties. Consider N vectors in p-space. The first i ($i < p$) principal components define an i-dimensional subspace of the p-space for which the sum of squared deviations of the N vectors from the i-space is minimized. Thus, the i principal components define a "best-fitting" i-space for the N p-variate vectors in the sense of defining the i-space with the smallest sum of squared deviations. One of the earliest uses of this method was in this context of a "fitting" procedure (Pearson, 1901). The two interpretations of principal components discussed will both be useful in understanding the present analysis.

This analysis falls logically into two stages. Steps 1 and 2 below form the first stage, and Steps 3, 4, and 5 comprise the second stage.

### Step 1: First Stage Principal Components Analysis

Consider an N x p matrix of p observations on each of N individuals. From this matrix of observations compute the p x p matrix of correlations[3] R. A principal components analysis is then performed on R yielding p characteristic roots and a p x p loading matrix[4] A.

If the N x p observation matrix standardized with respect to the sample means and variances, say Z, is considered by columns, each N x 1 column vector can be plotted in N-space. Then the correlation between variable i and variable j equals the cosine of the angle between the i-th and j-th column vectors of Z (Harman, 1960, p. 62). The i-th row of the loading matrix A can be thought of as a new representation of the i-th column of Z in the p-space of the principal component axes. Moreover, it can be shown that the angles between the rows of A are the same as the angles between the columns of Z in N-space. Similarly, it can be shown that the row vectors of A are of unit length as are the column vectors of Z. Thus, the points in p-space maintain precisely the same relationship to one another held by the original column vectors of Z in N-space.

The j-th element of the i-th row vector of A gives the length of the projection of the i-th variable vector on the j-th principal component axis. The same i,j element of A also represents the correlation between the i-th variable and the j-th component axis.

### Step 2: Location of Individuals in the Space of the Principal Components

It is common in principal components analysis to locate the original observation vectors (the rows of Z) in the space of the principal components. From the first stage principal components analysis we know that the j-th row of Z, say $\underline{z}'_j$, can be expressed as a linear combination of the scores on the component axes. Thus, if $\underline{f}_j$ represents these

[3]Use of the correlation matrix here rather than the covariance matrix is essentially equivalent to the assumption that the variables have been standardized with respect to their sample variances as well as means. While this may in fact sacrifice useful information contained in the relative magnitudes of the variances, in the social sciences such information is often of little value. Moreover, the standardization of the variables has a beneficial effect on their behavior under this analysis which will be noted.

[4]If R is essentially of rank q ($q < p$), then there are $p - q$ dimensions in which the variability is so small as to be negligible for our purposes. Therefore, in such cases, we consider the components corresponding to the q characteristic roots judged different from zero and the p x q loading matrix. This option is used only for those cases in which further computations would be difficult because of the deficiency of rank. In more common situations, all p dimensions should be used.

component scores (Harman, 1960, p. 360),

$$\underset{p \times 1}{\underline{z}_j} = \underset{p \times p \times 1}{A\,\underline{f}_j} . \quad (1)$$

Then $\underline{f}_j$ may be found by computing

$$\underline{f}_j = A^{-1}\,\underline{z}_j , \quad (2)$$

when A is of rank p. When the rank of A is essentially $q < p$ (cf. footnote 4), A is taken to be a $p \times q$ matrix and we make the working definition,

$$\underline{f}_j = (A'A)^{-1} A'\underline{z}_j . \quad (3)$$

Since A'A is the diagonal matrix of the characteristic roots, equation (3) gives an easy computational method for either the full or deficient rank case and will be used throughout the remainder of the paper.

The scores $\underline{f}_j$ then locate an individual observation in the p-space of the principal components, the same p-space in which the variables are located. Since both variables and individuals are plotted in the same p-space, it is important that the meaning of this operation be made clear as variables and individuals are rather different things.

One way to understand the results of such a combination is to ask: "What individual score vector $\underline{z}$ would have a corresponding component score vector $\underline{f}$ equal to a given variable point in the p-space?" In other words, recalling that the point in p-space representing the i-th variable is just the i-th row of A or equivalently the i-th column of A', say $\underline{a}_i$, we want to find the vector $\underline{z}^*_i$ such that

$$\underline{z}^*_i = A\,\underline{a}_i . \quad (4)$$

But by the rules of matrix multiplication, $\underline{z}^*_i$ is the i-th column of AA' which is known to equal R. That is to say

$$\underline{z}^*_i = \underline{r}_i , \quad (5)$$

the i-th column of R.

That a variable corresponds to a $\underline{z}$ score equal to a column of R has some intuitive justification. A score on variable j would be expected to lead to a similar score on variable k if j and k are highly correlated. Thus, if the score on variable j is one, the score on variable k equal to $r_{jk}$ is expected simply because of the relation between the variables.

It will be convenient later in the paper if we also find the $\underline{z}$ corresponding to the centroid of the variables in p-space, the mean of the rows of A. For $\underline{a}'_i$, the i-th row of A,

$$\bar{\underline{a}} = (1/p) \sum_{i=1}^{p} \underline{a}_i . \quad (6)$$

Then $\underline{z}^*_m$, the $\underline{z}$ score corresponding to the mean, is given by

$$\begin{aligned} \underline{z}^*_m &= A\,\bar{\underline{a}} \\ &= (1/p) \sum_{i=1}^{p} A\,\underline{a}_i \\ &= (1/p) \sum_{i=1}^{p} \underline{r}_i \\ &= \bar{\underline{r}} . \end{aligned} \quad (7)$$

Thus, the $\underline{z}$ score corresponding to the centroid of the variable vectors is the average of the rows or columns of the correlation matrix.

**Step 3: Second Stage Principal Components Analysis**

Step 3 begins the second stage of the analysis of spatial configuration. In stage one the variables and individual observations were located in the p-space of the first stage principal components. Now in stage two the problem is to reduce the dimensionality of the p-space to a smaller space in which the relationships among the variables may be more easily understood. Note that we are not discussing the reduction of dimensionality of the original observations as that could have been accomplished, if desired, in the first stage of the analysis. This second stage deals only with the scatter of the variable points about the component axes.

To accomplish a dimension reduction of the variability of the variable points, we first compute the covariance matrix, S, of the p component dimensions over the p variables,

$$S = (1/p)A'A - \bar{\underline{a}}\,\bar{\underline{a}}' . \quad (8)$$

From the characteristic roots and loading matrix of the second analysis, it can be determined if the variable points in p-space lie predominantly in a smaller subspace. If the first k principal components account for most of the variation of the points, then the space of the variables can be reduced to a k-space. In other words, if the last p − k roots are small, this indicates that the points deviate little from the k-dimensional space spanned by the first k vectors. As noted above, this reduction in dimensionality occurs not in the original data but in the dimensions in which the p variables differ from each other.

**Step 4: Projection of the Variables onto a Smaller Subspace**

Suppose in Step 3 it was found that k dimensions account for most of the variation of the p variables in p-space. Then the p variable points in p-space can be projected onto the k-dimensioned subspace as follows. Let B be the p x k matrix of loadings from Step 3, and let $A_*$ be the matrix A standardized for the mean $\underline{\bar{a}}$ defined in (6). Then

$$A_* = A - \underline{1}\,\underline{\bar{a}}' \qquad (9)$$

and

$$\underset{p \times p}{A_*} = \underset{p \times k \times p}{B\ H}\ , \qquad (10)$$

where $H = [\underline{h}_1 \vdots \underline{h}_2 \vdots \ldots \vdots \underline{h}_p]$ is the k x p matrix whose columns, $\underline{h}_i$, are the vectors of component scores of the i-th variable. H then gives the projection of the p points onto the k-space and can be found by

$$\underset{k \times p}{H} = \underset{k \times p \times k \times p \times p}{(B'B)^{-1} B'A'_*}\ . \qquad (11)$$

It often conveniently occurs that k is less than or equal to two. In such cases, the variables can be plotted and visually related to each other on a plane or line.

**Step 5: Projection of the Original Observations onto the Subspace of the Variables**

In Step 2 the observation vector $\underline{z}$ was located in the p-space of the principal components by computing the component scores $\underline{f}$. The vector $\underline{f}$, after being standardized by the mean $\underline{\bar{a}}$, can be projected onto the k-space of the variables as $A'_*$ is projected in equation (11). The two projections, first from the p-space of the original variables to the p-space of the principal components then to the k-space of the variables, can be combined into one step. For $\underline{g}_j$ the projection of observation j on the plane,

$$\begin{aligned} \underline{g}_j &= (B'B)^{-1} B'(\underline{f}_j - \underline{\bar{a}}) \qquad (12) \\ &= (B'B)^{-1} B'(A'A)^{-1} A'(\underline{z}_j - \underline{\bar{r}})\ , j = 1, \ldots, N. \end{aligned}$$

Note that the projection matrix acts on $(\underline{z} - \underline{\bar{r}})$ in (12). The effect this standardization on $\underline{\bar{r}}$ has in the analysis is that $\underline{\bar{r}}$ rather than $\underline{0}$ (the mean of the $\underline{z}$'s) becomes the baseline for comparisons. The elements of $\underline{z}$ are related to each other in this analysis according to their relative distances from $\underline{\bar{r}}$. The intuitive justification for this role of $\underline{\bar{r}}$ parallels that given on page 3 to justify the fact that $\underline{r}$ corresponds to the $\underline{z}$ score of a variable.

It should be noted that individual observations may show considerable variation about the k-space of the p variables. Individual observation vectors projected onto the k-space relate the individual to the primary differential dimensions of the variables considered.

**Relation of the Configural Analysis to Other Techniques**

Although principal components is sometimes used as a method of factor analysis, the use of principal components in this paper is not for the purpose of finding factors in the usual sense. Rather, the purpose of this analysis is more akin to the purpose of multidimensional scaling (Torgerson, 1958). In scaling there is concern with nonmetric data and experimental methods for finding distances between objects or stimuli with which we have had no concern. However, our first stage analysis provides locations of the variables in a convenient space and consequently distances between them, and our second stage analysis is similar to the second step of multidimensional scaling in which a distance matrix is factored.

In spite of the different motivations, there are aspects of factor analysis which relate to the analysis presented here. It has been common in factor analysis to plot variables on the factors computed, often as a step leading to a rotation of factor axes (Thurstone, 1947; Thomson, 1951; Guilford, 1954; and others). In addition, one specific method called the method of extended vectors treats test vectors in the space of the factor axes by extending the vectors to a plane perpendicular to the first factor, or on a unit sphere (Guilford, 1954, p. 514). The location of the variable points on this plane or sphere will often be similar to the configuration of variables achieved in our analysis.

A second factor analysis procedure bears some resemblance to our method though its purpose is entirely different. This is the so-called second order factoring. Second order factoring (Thurstone, 1947; Guilford, 1954) is the procedure of factoring the correlation matrix of nonorthogonal factors with the purpose of discovering more basic dimensions, often in search of a general factor. At the second stage of our analysis we factor the covariance matrix of the components computed only over the p variables, and the second stage principal components analysis is used strictly as a procedure to fit a smaller space to the points, not to discover any factors.

One offshoot of factor analysis and scaling which bears certain resemblances to this method is the work of Guttman on the radex (Guttman, 1954, 1965). Guttman is concerned with relationships among variables—specifically mental tests—in the form of linear or circular relationships. Something like Guttman's simplex or circumplex ordering of variables is often the result of our configural analysis. Guttman's smallest space analysis (Guttman, 1968) is a nonmetric approach definitely in the spirit of our procedure.

## An Application of the Configural Analysis to Measures of Colleges

In his book, *Who Goes Where to College?*, Astin (1965) presented student input data for 1,015 four-year colleges and universities. After computing factors for a sample of colleges with extensive data available, Astin used public sources of data to estimate the same factors for the 1,015 institutions. The five estimated student input factors were given the following names and interpretations by Astin (1965, pp. 54-55):

1. Intellectualism (INT). An entering student body with a high score would be expected to be high in academic aptitude (especially mathematical aptitude) and to have a high percentage of its students pursuing careers in science and planning to go on for the Ph.D. degrees.

2. Estheticism (EST). An entering student body with a high score would tend to have a high percentage both of students who achieved in literature and art during high school and of students who aspire to careers in these fields.

3. Status (STA). An entering student body with a high score would be expected to have a high percentage of students who come from high socioeconomic backgrounds and who themselves aspire to careers in Enterprising fields (lawyers, business executives, politicians).

4. Pragmatism (PRA). An entering student body with a high score would tend to have a high percentage of students planning careers in Realistic fields (engineering, agriculture, physical education) and a low percentage of students planning careers in Social fields (teaching, sociology, psychology, nursing).

5. Masculinity (MAS). An entering student body with a high score would tend to have a high percentage of men, a high percentage of students seeking professional degrees (LL.B., M.D., D.D.S.), and a low percentage of students planning careers in Social fields.

These five variables will be used here in an analysis of spatial configuration.

**Step 1**

The correlation matrix of the five variables for the 1,013 colleges with complete data is presented in Table 1.

**Table 1**

**Intercorrelations of Astin Variables[5]**

| | *INT* | *EST* | *STA* | *PRA* | *MAS* |
|---|---|---|---|---|---|
| INT | 1.00 | | | | |
| EST | .34 | 1.00 | | | |
| STA | .54 | .18 | 1.00 | | |
| PRA | .43 | −.34 | .10 | 1.00 | |
| MAS | .38 | −.46 | .43 | .61 | 1.00 |

The variables were given by Astin in a standardized form with mean of 50 and standard deviation of 10, but all references to them here will be in an alternate standardized form with mean zero and standard deviation of one.

A principal components analysis was performed on R and the results are given in Table 2.

**Table 2**

**Principal Components Analysis of the Correlation Matrix of Astin Variables**

| Components: | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Roots: | 2.281 | 1.591 | 0.711 | 0.220 | 0.197 |
| Percent Trace: | 45.6 | 31.8 | 14.2 | 4.4 | 3.9 |
| Loadings: | | | | | |
| INT | .7110 | .5640 | .2862 | −.2411 | −.1908 |
| EST | −.2390 | .9002 | .2399 | .0649 | .2662 |
| STA | .6174 | .5185 | −.5429 | .2183 | −.0870 |
| PRA | .7638 | −.3256 | .4837 | .2766 | .0062 |
| MAS | .8680 | −.2965 | −.2079 | −.1834 | .2861 |
| Mean | .5442 | .2721 | .0518 | .0271 | .0561 |

The rows of A, the loading matrix in Table 2, locate the variables in five-space.

**Step 2**

Individual observations can be located in the five-space by computing $\underline{f}$ in equation (3).

[5]The correlation matrix given in Table 1 does not exactly agree with that given by Astin (1965, p. 50). The reason for the discrepancy is not known. However, the analysis when performed on Astin's matrix yielded essentially the same results reported here.

## Step 3

The covariance matrix, S, of the five components over the five variables is

$$S = \begin{bmatrix} .1599 & & & & \\ -.1481 & .2442 & & & \\ -.0282 & -.0141 & .1396 & & \\ -.0147 & -.0074 & -.0014 & .0433 & \\ -.0305 & -.0153 & -.0029 & -.0028 & .0362 \end{bmatrix}. \quad (13)$$

Table 3 gives the results of a principal components analysis on S. The first two dimensions in the second stage analysis account for 81.2% of the trace. Thus, we know that the deviations of the variable points from their centroid is almost contained in a space of two dimensions (i.e., a plane). Because of the value of providing a visual representation, we may let k = 2.

Table 3

Second Stage Principal Components Analysis

| Components: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Roots: | 0.356 | 0.149 | 0.073 | 0.044 |
| Percent Trace: | 57.2 | 24.0 | 11.7 | 7.1 |
| Loadings: | | | | |
| 1 | −.3621 | −.0909 | .1433 | .0047 |
| 2 | .4742 | −.0826 | .1120 | .0035 |
| 3 | .0161 | .3655 | .0756 | .0019 |
| 4 | .0057 | .0128 | −.0864 | .1888 |
| 5 | .0117 | .0259 | −.1640 | −.0921 |

## Step 4

For B, the 5 x 2 portion of the loading matrix outlined in Table 3,

$$(B'B)^{-1}B' = \begin{bmatrix} -1.0161 & 1.3306 & 0.0451 & 0.0160 & 0.0329 \\ -0.6077 & -0.5522 & 2.4446 & 0.0857 & 0.1733 \end{bmatrix}. \quad (14)$$

Then the locations of the five variables in two-space can be computed by equation (11). The result, H, is given below.

$$H = \begin{array}{c} \begin{array}{ccccc} \text{INT} & \text{EST} & \text{STA} & \text{PRA} & \text{MAS} \end{array} \\ \begin{bmatrix} 0.2171 & 1.6475 & 0.2251 & -0.9966 & -1.0931 \\ 0.2447 & 0.6287 & -1.6427 & 1.2652 & -0.4958 \end{bmatrix} \end{array} \quad (15)$$

The variables can be plotted on the plane to obtain a pictorial representation of the relationships among them. Figure 1 gives that representation.

## Step 5

The projection matrix, say P, for locating an individual observation vector (a college) on the plane is computed as in equation (12).

$$P = (B'B)^{-1}B'(A'A)^{-1}A'$$

$$= \begin{bmatrix} 0.1218 & 0.9246 & 0.1264 & -0.5593 & -0.6135 \\ 0.3272 & 0.8409 & -2.1972 & 1.6922 & -0.6631 \end{bmatrix}. \quad (16)$$

Premultiplying by P a college's vector of scores (or a college group mean vector) standardized by $\underline{T}$ (cf. equation (12)) gives the college's (or group's) location on the plane.

**The Variables**

First, from H in equation (15), the five Astin variables were plotted on a plane as shown by the points in Figure 1. Recall that this representation is the projection of the deviations of the variable vectors of unit length (in p-space) from their mean onto the two-dimensional subspace which minimizes the deviation of the variable points around it. In this case the plane accounted for 81.2% of the variation of the variable points.

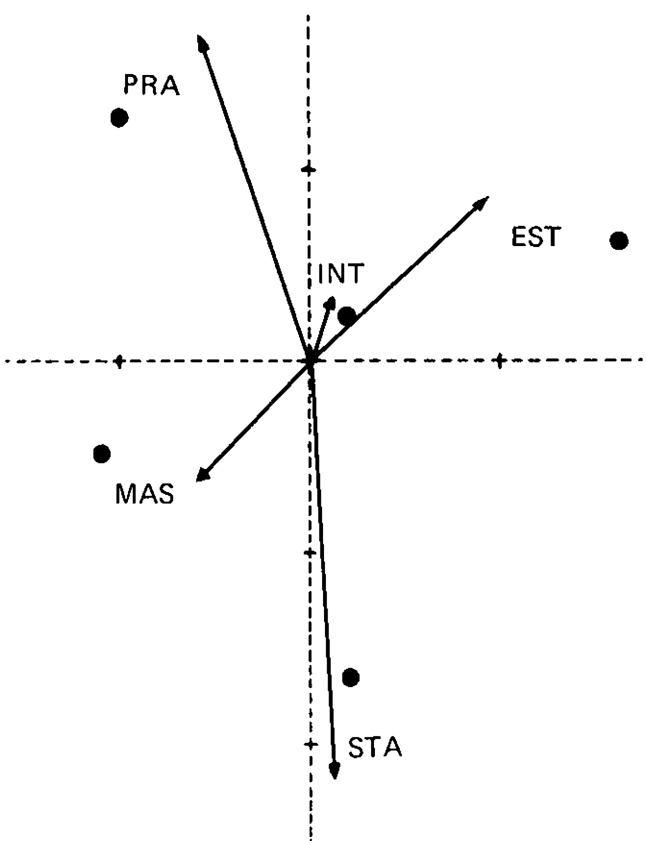



**Figure 1. Location of variables and their forces on a plane.**

The variable points in Figure 1 give information about how variables in a vector or profile relate to each other. The distances between the variable points as specified in the five-space make geometrically precise the idea of distance implied in the correlation matrix as a measure of relatedness. The reduction to the plane retains those aspects of the distances accounted for by the variability in the two dimensions. Thus, to the degree that the variable points fit the plane, as measured by the portion of the trace corresponding to the two dimensions, highly correlated variables will be represented by proximal points and conversely.

A second description of the relationship of the variables can be obtained from the projection matrix P in equation (16). By examination of the equation

$$\underline{g}_j = P\,(\underline{z}_j - \underline{\bar{r}}) \qquad (17)$$

it is seen that the location of an individual's point $\underline{g}_j$ on the plane will be a weighted sum of the columns of P in which the weights are the elements of the difference vector $(\underline{z}_j - \underline{\bar{r}})$. Thus, the columns of P, indicated by arrows in Figure 1, may be thought of as forces whose weighted resolution (equation (17)) locates an individual in the plane.

The location of Intellectualism (INT) near the center of the plane and with a very small force in the projection matrix means that the INT score of a college, compared with its other scores, has a relatively small effect in differentiating colleges in this plane. Thus, when regions in the plane near variable points are considered as relative orientations which colleges may have, the other four variables are more dominant as differentiators than the INT variable.

This relatively small importance of the INT variable illustrates one of the peculiar aspects of this form of analysis. The analysis will identify a variable such as INT which does not contribute to the locating of colleges in orientation regions because it is nearly equally correlated with variables which define the regions. The other side of the same coin is the fact that the analysis does *not* reflect differences between colleges on that variable, a point to be remembered in subsequent comparisons between colleges. Thus, on the one hand the analysis reveals the fact that colleges with each of the other four orientations may be either high or low in INT and that no orientation is systematically higher or lower than others. That is to say, INT, with the strong influence of academic aptitude and educational plans, cuts across other orientation variables. On the other hand, the magnitude of the INT variable, which may be of interest in itself, is largely ignored in this analysis.

**Colleges, some examples**

Given the limitation of the analysis for this particular data, let us consider what the analysis does tell us about colleges' orientations. We will do this by comparing the discussion of college profiles and conclusions of Astin with our analysis and conclusions.

Consider first two profiles which Astin (1965, p. 86) used to illustrate his data, those of Rice University and Princeton University. An adaptation of the profiles is reproduced in Figure 2 for the five variables under consideration. Astin discussed three kinds of interpretation that can be made: (1) comparison of scores within an institutional profile; (2) comparison with institutions in general; and (3) comparison of specific institutions with each other. Each type of interpretation has a parallel based on the configural analysis, but as will be seen there are definite differences between the types of conclusions that can be drawn from the two types of analysis.

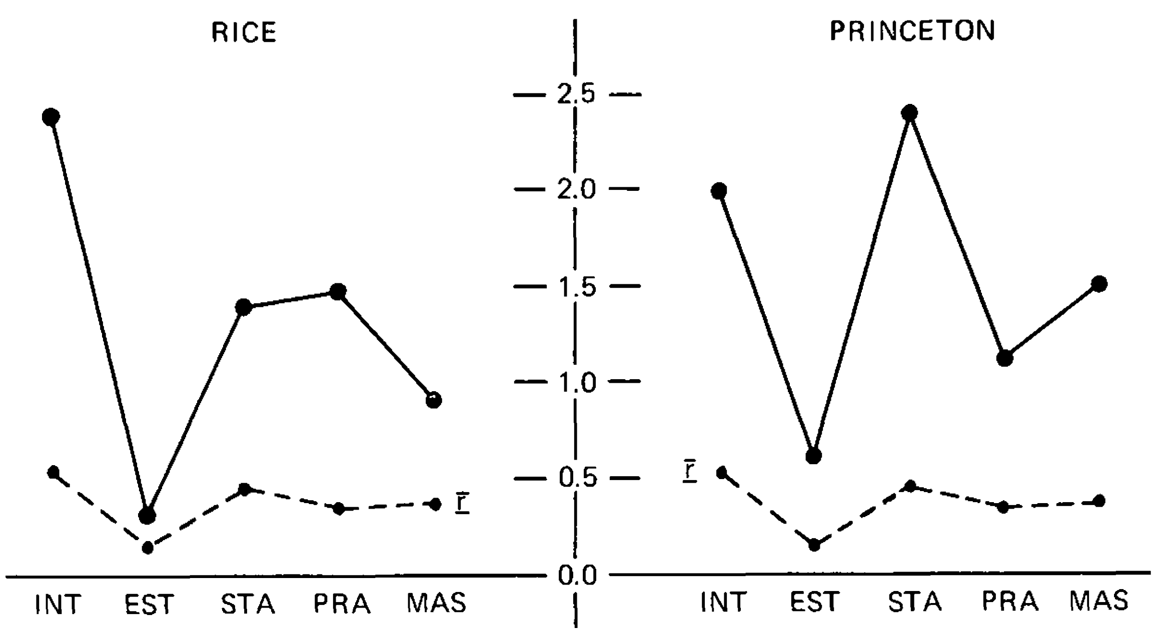



**Figure 2. Profile of two colleges on five Astin variables.**

**Comparison 1.** Astin's comparisons of scores within institutional profiles were made on a variable by variable basis reflecting the common difficulty in interpreting profile scores. Thus, he found that entering students at Rice were much lower on Estheticism than on Intellectualism. Princeton students were similarly lower on Estheticism than other orientations and highest on Status and Intellectualism. By contrast. as shown in Figure 3, the configural analysis shows each college profile as a single point whose location in the plane summarizes the important characteristics of the profile. Thus, the location of Rice shows the resolution of the similarly dominant STA and PRA variables with slightly more weight on MAS than EST. Princeton, on the other hand, which was dominated by STA, is pulled strongly in that direction. The comparison of scores within a profile by the configural analysis results in a summary or resolution of the profile in a single point rather than requiring variable by variable comparisons.

**Comparison 2.** To better understand the way Astin's second type of comparison relates to the configural analysis there are several facts that should be considered. First, in Astin's analysis the institutions formed a population with mean of zero on all scales. In the configural analysis an institution with a perfectly average score ($\underline{z} = \underline{0}$) will not fall exactly at the origin in the plane, but will appear at a point near it as indicated by the square in Figure 3. In fact, the centroid of the variable points in p-space, a point which has been shown to correspond to a score profile of $\underline{z} = \underline{\bar{r}}$, is a point that maps into the origin in the plane. Moreover, it can be shown that all profiles parallel to any given profile fall at the same point on the plane. Therefore, all profiles parallel to $\underline{\bar{r}}$ fall at the origin. Note that this means that it is the shape, not the level, of the profile which is preserved in the transformation to the plane.

In the second type of comparison, both Rice and Princeton were above the mean on all variables, a fact that would be noted by Astin's analysis of the profiles but not by ours. Thus, if the overall level of the profile is of interest, Astin's comparison is definitely preferable. Instead, the configural analysis relates the relative orientation of a particular college to the mean of all colleges by how close to each other they are on the plane. Princeton showed a degree of dominance of the STA orientation which is quite unlike colleges in general. On the other hand, Rice lies nearer the mean for all colleges and demonstrates a balanced orientation. It should be noted that the balanced orientation mentioned may result from several types of "balance." The mean for all colleges lies near the center of the plane and all elements of $(\underline{\bar{z}} - \underline{\bar{r}})$ are similar and in this sense balanced. Rice, on the other hand, lies near the center of the plane more because of the resolution of the PRA and STA forces in opposite directions than because of a flat profile, illustrating another kind of balance indistinguishable in the planar configuration from the first-mentioned kind. In addition, Astin noted that both Rice and Princeton recruit students very high in academic aptitude and scientific orientation and therefore have high INT scores. As already discussed this effect is also largely ignored in our analysis because of the small size of the INT force vector.

**Comparison 3.** In comparing Rice with Princeton, Astin noted the great similarities between the two schools. The principle difference was that Princeton was higher on STA. Since the configural analysis concentrates on relative orientations rather than level of the variables, this difference on the STA variable becomes more important in comparing the two institutions on the plane. Thus, the difference in orientations is primarily that Princeton has much greater relative orientation to Enterprising fields and high socioeconomic status.

Thus, in the three types of comparisons there are both advantages and disadvantages to each method. The two methods have different purposes and lead to different results. The purpose determines the appropriateness or inappropriateness of the method to be used. In this study we avow an interest in the relative orientations of colleges to different aspects of the educational enterprise. In the coincidence of that purpose with the purpose of the configural analysis its appropriateness is assured. For other purposes other methods would be more appropriate.

The locations of several other colleges have also been presented in Figure 3. The large state universities and technical institutes, located in the upper left quadrant, demonstrate the relative orientation of these institutions in Realistic (engineer-

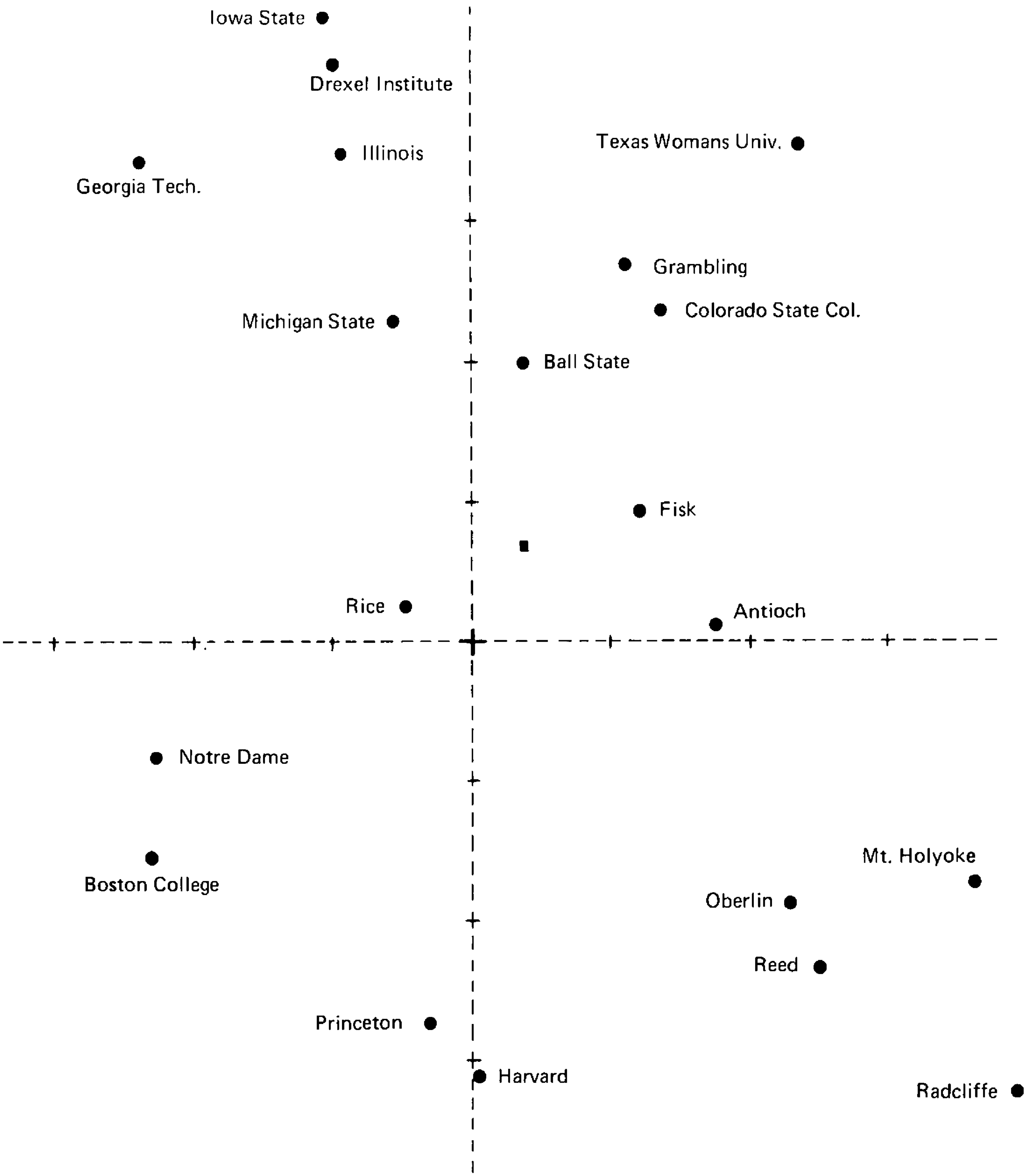
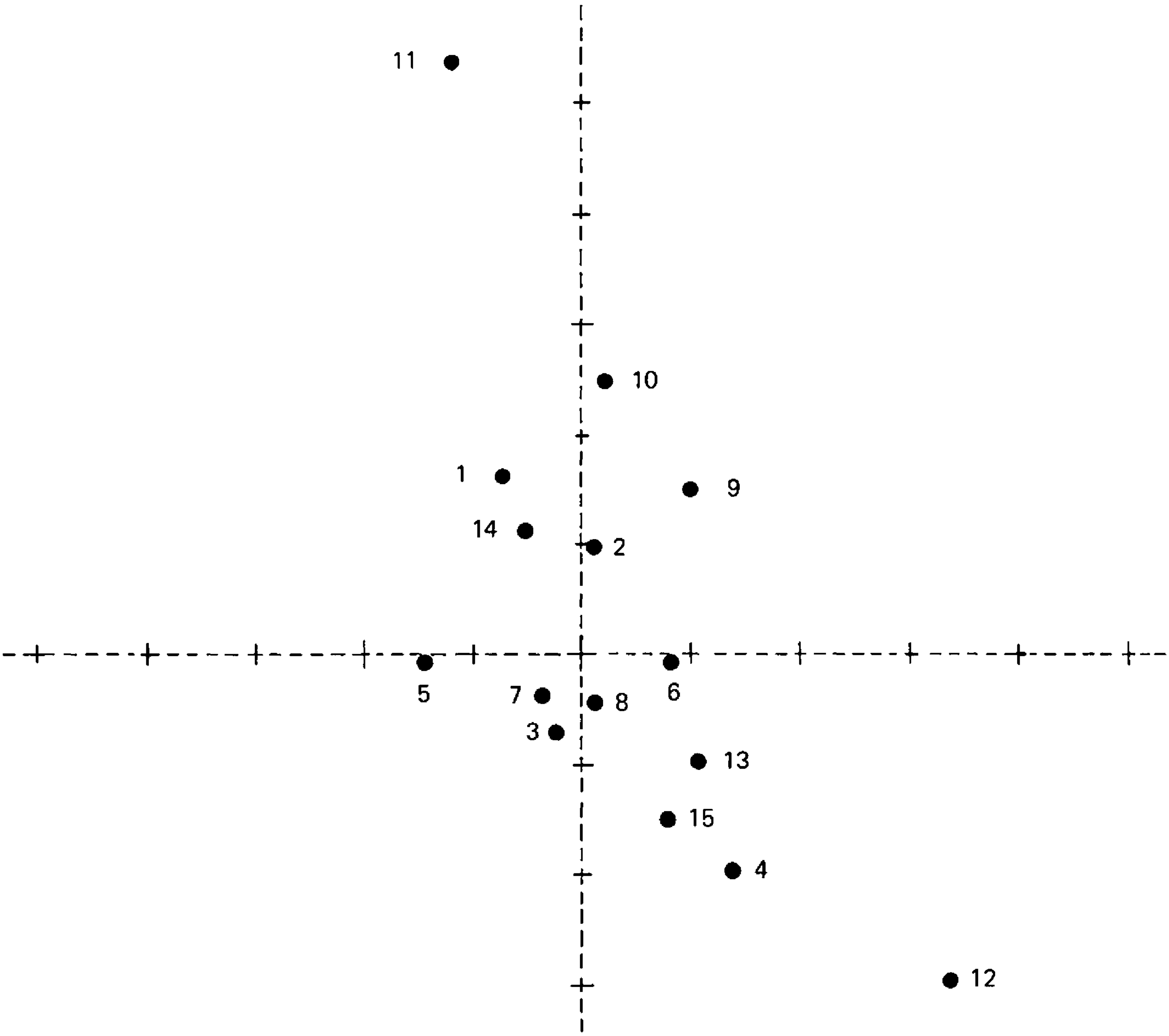



**Figure 3. Location of colleges on the plane.**




1. Public universities (109)
2. Public liberal arts colleges (137)
3. Private nonsectarian univ. (46)
4. Private nonsect. lib. arts col. (89)
5. Catholic universities (26)
6. Catholic lib. arts col. (153)
7. Protestant universities (28)
8. Protestant lib. arts col. (231)
9. Teacher colleges (42)
10. Predominantly Negro col. (59)
11. Technical institutes (30)
12. Art institutes (10)
13. Small colleges* (41)
14. Large colleges* (59)
15. Selective colleges* (56)

*Small colleges were defined as those with a score less than 35 on Astin's size index. Large colleges were those with a score greater than 65. Selective colleges were those with a score over 65 on Astin's selectivity index.

**Figure 4. Location of groups of colleges on a plane.**

ing, etc.) rather than Social fields. As public institutions they lack a strong Status domination, have some Masculinity pull, and as a result are located generally to the upper left. Teacher colleges and some former teacher colleges are located to the upper right, reflecting the relative strength of their Social and Artistic orientations. Note that two predominantly Negro colleges fall in this quadrant also, suggesting a similarity in orientation with the teacher colleges. Several small, elite private women's colleges and artistically-oriented liberal arts colleges are located in the lower right quadrant. These colleges show STA and EST orientations. Elite private universities with more MAS pull and less Social and EST pull are to the lower left.

### Types of Colleges

The observations made above on the basis of the sample of institutions in Figure 3 can be made more systematically by computing means for groups of colleges of the same type. Fifteen types of colleges were considered and these types are listed in Figure 4 along with the number of colleges included in each group.

The locations of college type means in Figure 4 support the observations made about where types of colleges fall. It is interesting to note the strong STA orientation (high SES and Enterprising emphasis) in the most selective and smallest colleges. Of course, this result comes as no surprise. It is mainly the public institutions which offset the Status domination in favor of a heavy Pragmatic orientation with emphasis on Realistic vocational choices in the universities and Social and Artistic emphasis in the teacher colleges. Thus, clearly the public institutions in this country have more egalitarian orientations than the private colleges as well as technical and social orientations rather than business orientations.

One final point to be made is that there seem to be consistent and meaningful distinctions which can be made about colleges on the basis of this analysis of spatial configuration. Recall that these distinctions have made very little use of the differences in academic ability of the students. Thus, it seems that one can discuss the diversity of orientations in American colleges and universities in this framework with no reference to or implication of corresponding differences in academic ability of the students.

## Other Applications

One important group of instruments for which the analysis of spatial configuration seems especially suited is the interest inventories. Here a vector of scores is interpreted in terms of the relative orientation of the individual to different interest patterns. Often measures of absolute degree of interest in any area covered by the instrument are complicated by such things as response sets and are therefore not of greatest importance.

A configural analysis of Holland's Vocational Preference Inventory (Cole, Whitney, and Holland, in press) has been performed. The analysis was helpful in relating the VPI variables to each other, in "typefying" an individual by his location on the plane, and in locating occupational groups on the plane.

Because of the way the configural analysis concerns the dimensions on which variables differ, it seems especially appropriate as a way to study the relationships of scales in an instrument. In a paper concerned with differential validity in a battery of tests, it was informative to consider the pattern of the test variables on the second and third principal components (Cole, 1969). A similar configuration would be obtained by this analysis. When variables are too close together they may be measuring too similar a concept. Areas may appear in which a measure is called for but has not been included.

One interesting possibility is to use the analysis at the level of scale construction in order to discover the dimensions on which the items on a scale differ.[6] Then one can judge which dimensions of item differences seem to be important ones and which are to be eliminated. When two scales are closely related, items from both may be analyzed simultaneously in order to discover overlapping items responsible for the similarity.

[6]Gary R. Hanson suggested this use of the configural analysis for item analysis. Preliminary analyses by Hanson suggest that the analysis can be very useful in this way.

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